Suggested Solutions to: Resit Exam, Spring 2015 Industrial Organization August 13, 2015

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Question 1: Can price signal quality?

Part (a)

It is stated in the question that the payoff of a consumer with taste parameter θ is

$$\begin{cases} \theta + s - p & \text{if buying} \\ 0 & \text{if not buying.} \end{cases}$$

This means that a consumer is indfferent between buying and not if, and only if

$$\theta + s - p = 0 \Leftrightarrow \theta = p - s \equiv \widehat{\theta}.$$

Therefore, all consumer with $\theta > \widehat{\theta}$ will buy and those with $\theta < \widehat{\theta}$ will not buy. The total mass of consumers who want to buy, given a price p, is thus given by the mass of consumers with a taste parameter satisfying $\theta \in \left[\widehat{\theta}, 1\right]$. By assumption the θ's take values between zero and one, and the total mass of consumers equals one. Therefore, if $\widehat{\theta} \leq 0$ (or, equivalently, $p \leq s$), then all consumers want to buy: demand is one — the first line in the stated demand function. If $\widehat{\theta} \geq 1$ (or, equivalently, $p \geq 1+s$), then no consumer wants to buy: demand is zero — the third line in the stated demand function. It was also assumed that the distribution of taste parameters was uniform on $[0, 1]$. Therefore, if $\widehat{\theta} \in \left(\widehat{\theta}, 1\right)$ (or, equivalently, $s < p < 1 + s$), then demand is

$$\begin{aligned} 1 - \widehat{\theta} &= 1 - (p - s) \\ &= 1 + s - p, \end{aligned}$$

which is the middle line in the demand function stated in the question.

The firm's optimally chosen price must solve the following problem:

$$\max_p (p - c) Q(p).$$

Consider first the possibility of an optimum with $p \in (s, 1 + s)$. Then the problem can be written as

$$\max_p (p - c)(1 + s - p) \tag{1}$$

and the first-order condition is $1+s-p-(p-c) = 0$ or

$$p^* = \frac{1 + s + c}{2}.$$

With the help of some simple algebra one can verify that, indeed, $p^* \in (s, 1 + s)$.[1] Therefore, since the second-order condition also is fine, p^* must be the optimal price. Moreover, plugging p^* back into the objective in (1) yields (the students should show the algebra)

$$\pi^* = \frac{(1 + s - c)^2}{4}.$$

Part (b)

The first necessary condition, IC-bad, says that the bad type must not want to mimic the good type; that is, the bad type must prefer to charge the price p_b^*, and thus be acknowledged by all customers as a low-quality firm, rather than to charge the good type's price p_g and thus be perceived by all the non-informed costumers as a high-quality firm. The condition IC-bad can be stated formally as follows:

$$\pi_b^* \geq \widehat{\pi}_b(p_g), \tag{IC-bad}$$

where π_b^* is the bad type's profit if charging p_b^* and $\widehat{\pi}_b(p_g)$ is the bad type's profit if charging p_g. The first profit expression, which is also defined in the question, is given by

$$\pi_b^* \equiv \frac{(1 + s_b)^2}{4},$$

[1] We must then use the assumptions that $c < 1$ and $s < 1$.

and the latter profit expression is defined by

$$\widehat{\pi}_b(p_g) \equiv p_g [1 + \lambda s_b + (1-\lambda) s_g - p_g].$$

The expression in square brackets is the firm's demand when charging p_g. The second and third terms in this expression reflect the facts that a fraction λ of the customers can observe the true quality and therefore knows that $s = s_b$, whereas a fraction $1-\lambda$ are uninformed and thus erroneously conclude that $s = s_g$.

The second necessary condition, IC-good, says that the good type must prefer to charge the price p_g, and thus be perceived as a high-quality firm, to choosing the price that maximizes its profit — given that it is perceived by all the non-informed costumers as a low-quality firm. The condition IC-good can be stated formally as follows:

$$\widehat{\pi}_g(p_g) \geq \pi_g^*. \qquad \text{(IC-good)}$$

Here, $\widehat{\pi}_g(p_g) \equiv p_g(1 + s_g - p_g)$ is the good type's profit if charging p_g and being perceived as a high-quality firm. The right-hand side of IC-good is the firm's profit if choosing the best price among all $p \neq p_g$ and being perceived as a low-quality firm by all non-informed customers. It is given by

$$\begin{aligned} \pi_g^* &\equiv \max_p p[1 + (1-\lambda) s_b + \lambda s_g - p] \\ &= \frac{[1 + (1-\lambda) s_b + \lambda s_g]^2}{4}. \end{aligned}$$

Part (c)

Now assume, as stated in the question, that $s_g = \frac{4}{5}$, $s_b = \frac{1}{5}$, and $\lambda = \frac{1}{2}$. Then IC-bad simplifies to

$$\frac{(1+s_b)^2}{4} \geq p_g [1 + \lambda s_b + (1-\lambda) s_g - p_g] \qquad \text{(IC-bad)}$$

or

$$\frac{\left(1+\frac{1}{5}\right)^2}{4} \geq p_g \left[1 + \frac{1}{2} * \frac{1}{5} + \frac{1}{2} * \frac{4}{5} - p_g\right] \qquad \text{(IC-bad)}$$

or

$$\frac{9}{25} \geq p_g \left(\frac{3}{2} - p_g\right). \qquad \text{(IC-bad)}$$

The figure in the question indeed shows this condition: the left-hand side is represented by the lower flat line (which is also indicated by "π_b^*"); the right-hand side is represented by the lower curve (which crosses the horizontal axis at the origin and at 1.5). The condition is satisfied for values of p_g such that the flat line is at or above the curve — so for values of p_g that are sufficiently low or sufficiently high, as indicated in the figure below.

Similarly, IC-good simplifies to

$$p_g(1 + s_g - p_g) \geq \frac{[1 + (1-\lambda) s_b + \lambda s_g]^2}{4} \qquad \text{(IC-good)}$$

or

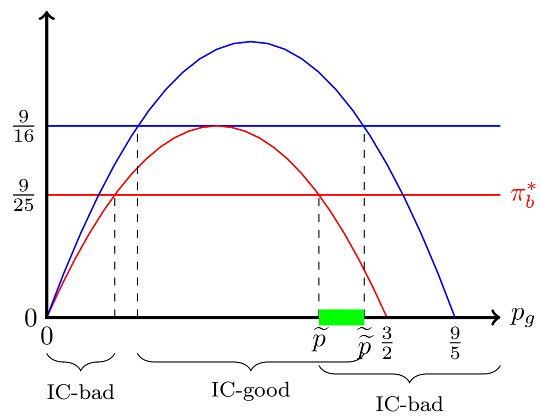
$$p_g\left(1 + \frac{4}{5} - p_g\right) \geq \frac{\left[1 + \frac{1}{2} \times \frac{1}{5} + \frac{1}{2} \times \frac{4}{5}\right]^2}{4} \qquad \text{(IC-good)}$$

or

$$p_g\left(\frac{9}{5} - p_g\right) \geq \frac{9}{16}. \qquad \text{(IC-good)}$$

Also this condition is illustrated by the figure in the question: the right-hand side is represented by the upper flat line; the left-hand side is represented by the upper curve (which crosses the horizontal axis at the origin and at $p_g = \frac{9}{5}$). The condition is satisfied for values of p_g such that the flat line is at or below the curve — so for values of p_g that lie in an intermediate range, as indicated in the figure below.

For p_g to be part of a separating equilibrium, both IC-bad and IC-good must be satisfied. This is the case for values of p_g in the green area in the figure — or, for any $p_g \in \left[\widetilde{p}, \widetilde{\widetilde{p}}\right]$.



Part (d)

(i) *What is meany by predatory pricing and limit pricing?* The idea: a (dominant) firm may start a price war in order to get rid of a competitor.

- If competitor is currently in the market: "**predatory pricing**."
- If competitor is a potential entrant: "**limit pricing**."

Ordover and Willig's (1981) definition of predatory pricing: "Predation is a response to a rival that sacrifices part of the profit that could be earned under competitive circumstances, were the rival to remain viable, in order to induce exit and gain consequent additional monopoly profit."

(ii) *Scholars at the University of Chicago have argued that predatory pricing cannot be rational (and, hence, it does not occur). Explain the reasoning behind this argument.*

- Predatory pricing is costly while you are doing it (a price war).
- Therefore, the initial price war must be followed by a period with high prices, so that the predator can recover the early losses.
- But the high prices will encourage entry again, either by the same firm or by a new entrant.
- Realizing that it will not be able to recover its losses, a firm will choose not to charge a predatory price in the first place.

(iii) How did Milgrom and Roberts (in Tirole's simplified version) model limit pricing? Focus on the key model assumptions and explain how the logic of the model works.

- The incumbent firm's cost is either low or a high,
  - But the potential entrant does not know the cost.
  - If the cost were actually low, then the entrant would not be able to compete profitably and therefore be better off not entering.
- The incumbent does not want the other firm to enter.
  - Therefore, the incumbent has an incentive to try to make the entrant believe it is a low-cost firm (regardless of whether this is true or not).
- The incumbent firm may be able to induce those beliefs in the entrant by charging a very low price early on.
  - The potential entrant, observing this price, might then infer that the incumbent must be a low-cost firm.
  - For only a low-cost firm would have an incentive to charge such a low price.

Question 2: A market with vertically related firms

Part (a)

Solve for the subgame-perfect equilibrium values of p, w and λ.

We solve the game by backward induction, first studying the downstream firm's problem at stage 2 and then the upstream firm's problem at stage 1.

The downstream firm's profits are:

$$\pi^D = \lambda (1-p)(p-w) - \frac{1}{2}\lambda^2. \tag{2}$$

The first-order conditions to the problem of maximizing these profits with respect to p and λ can be written as

$$\frac{\partial \pi^D}{\partial p} = \lambda \left[-(p-w) + (1-p)\right] = 0, \tag{3}$$

$$\frac{\partial \pi^D}{\partial \lambda} = (1-p)(p-w) - \lambda = 0. \tag{4}$$

Notice that $\lambda = 0$ cannot be profit-maximizing, for this would yield zero profits whereas setting both p and λ positive but small yields positive profits. Equation (3) therefore implies that

$$-(p-w) + (1-p) = 0 \Rightarrow \boxed{p^*(w) = \frac{1+w}{2}.} \tag{5}$$

And then equation (4) gives us

$$\lambda^*(w) = (1-p^*(w))(p^*(w)-w) = \boxed{\frac{(1-w)^2}{4}}. \tag{6}$$

The upstream firm's profits, given that it anticipates the downstream firm's optimal response, are:

$$\begin{aligned}
\pi^U &= \lambda^*(w)\left[1-p^*(w)\right](w-c) \\
&= \frac{(1-w)^2}{4}\left[1-\frac{1+w}{2}\right](w-c) \\
&= \frac{(1-w)^3 (w-c)}{8}.
\end{aligned}$$

The first-order condition is:

$$\frac{\partial \pi^U}{\partial w} = \frac{-3(1-w)^2(w-c) + (1-w)^3}{8} = 0. \tag{7}$$

Notice that $w = 1$ cannot be profit-maximizing, for this would yield zero profits whereas setting w positive but small yields positive profits. Equation (7) therefore implies that

$$3(w-c) = 1-w \Rightarrow w^* = \frac{1+3c}{4}.$$

This in turn yields

$$p^*(w^*) = \frac{1+w^*}{2} = \frac{1+\frac{1+3c}{4}}{2} = \frac{5+3c}{8}$$

and

$$\begin{aligned}
\lambda^*(w^*) = \frac{(1-w^*)^2}{4} = \frac{\left(1-\frac{1+3c}{4}\right)^2}{4} &= \frac{(3-3c)^2}{64} \\
&= \frac{9(1-c)^2}{64}.
\end{aligned}$$

Summing up, we have that the subgame-perfect equilibrium values of p, w and λ are:

$$p^* = \frac{5+3c}{8}, \quad w^* = \frac{1+3c}{4}, \quad \lambda^* = \frac{9(1-c)^2}{64}.$$

Part (b)

Suppose the firms integrate and become one single firm. Calculate again the subgame-perfect equilibrium values of p and λ.

The integrated firm must incur the production cost c for every unit that it is selling. It must also incur the advertising costs. The wholesale price w, however, does not matter at all under integration. The integrated firm's profits can therefore be written as:

$$\pi^I = \lambda(1-p)(p-c) - \frac{1}{2}\lambda^2.$$

Notice that this expression is identical to (2) above, except that the w in (2) is here replaced by c. That means that we can use the results in equations (5) and (6) above, only substituting c for w. We thus have that the subgame-perfect equilibrium values of p and λ are given by

$$p^I = \frac{1+c}{2}, \qquad \lambda^I = \frac{(1-c)^2}{4}.$$

Part (c)

Would you expect aggregate consumer surplus to be largest under integration or under non-integration? Spell out your reasons and the logic. Answer verbally only.

- We should expect aggregate consumer surplus to be larger under integration than under non-integration.
- The reason is that the actions taken by the non-integrated downstream firm influences also the upstream firm's profits. Moreover, internalizing those external effects (which the firms would do after integration) helps also the consumers, not only the upstream firm's profits. In particular, the integrated firm will have a stronger incentive to lower the price and to do advertising, since both the downstream and upstream profits are positively affected by that. Also, both activities help consumers and the consumer surplus (because consumers gain from a lower price and from the opportunity to buy the good).
- Because of the logic discussed above, we should expect that the retail price is lower under integration and the advertising level is higher under integration (one can confirm that they indeed are).

Part (d)

Suppose now that, as under (a), the firms are not integrated. Moreover, the retail price p is now chosen not by Firm D at stage 2, but by Firm U at stage 1 (we can interpret this as resale price maintenance, RPM). Everything else in the model is unchanged. Would you expect RPM, modeled like this, to give rise to the same outcome (i.e., the same equilibrium values of p and λ) as under integration? Spell out your reasons and the logic. Answer verbally only.

- No, we should not expect RPM (in this sense) to give rise to the same outcome as under integration.
- The reason is that, in the original model, there are *two* variables that are chosen by the downstream firm, which both involve an externality. Under RPM the upstream firm can exert full control over one of these (the retail price), but the advertising level is still chosen by the downstream firm. It is not clear how the upstream firm, with the help of a *single* instrument, would ensure that the downstream firm behaves correctly in *two* independent dimensions.